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Advances and Limitations of Level Set Methods for Modeling the Freshwater–Saltwater Interface in Coastal Aquifers: Insights from the Henry Problem

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Abstract
Coastal intrusion of salt water into freshwater aquifers is a major threat to the freshwater security in the world. This paper is a critical review of the level set technique as applied in monitoring and modeling the freshwater-saltwater interface during the coastal aquifer system, based on the case of Henry problem. Combined with high-order numerical discretization, and with a suitable reinitialisation approach, level set formulations have demonstrated the ability to accurately solve interface dynamics and achieve better geometric flexibility than traditional modeling methods. Nevertheless, there remain formidable mathematical problems such as numerical diffusion that affects the sharpness of the interface, inefficient maintenance of mass balance over long simulations and computational inefficiencies, especially in the three-dimensional and long-term modeling cases. The study suggests the hybrid methodological frameworks to help eliminate these limitations by using a combination of level set tracking with the volume-of-fluid corrections to improve mass conservation and make it more applicable to real-life field studies.

Introduction
Approximately forty percent of the world population resides within one hundred kilometers of the shoreline, which has resulted in high reliance on the ocean aquifer systems in the availability of fresh water. Nevertheless, the resources are increasingly vulnerable to salinization, where the heavy seawater is taking the place of or mixing with the less dense freshwater [1, 16]. This has been very much documented to a large geographical extent not only along the Mediterranean Basin but also in the Pacific Islands.

Saltwater intrusion is a process that deals with several factors. Natural factors are tidal changes and storm surges and also anthropogenic factors, especially the over-pumping of the ground water depleting

the fresh water discharge and lowering the water table. There is more complexity with climate change, which is forecasted to increase the sea levels and alter the pattern of precipitation [1].
Historical roots of saltwater intrusion modelling Saltwater intrusion modelling has its conceptual foundations in Ghyben (1888) and Herzberg (1901), who formulated equations of the relationship between the altitude of freshwater table and depth of the interface through hydrostatic conditions. The dynamics of the ground water flows were not provided in these early conceptions but were a fair first-order approximation [1]. Subsequently, Henry (1959) gave a pioneer steady-state solution to a simplified two-dimensional coastal aquifer that has been canonically used in the validation of numerical models [9]. Later formulations have included hydrodynamic concepts as it became realized that the location of interfaces is due to the combination of advective transport, dispersive mixing and density-driven convection [2, 7].

Problem Statement and Objectives.
Saltwater intrusion modelling has two traditional paradigms sharp-interface models and density-dependent models. The formulations of sharp interfaces define freshwater and salty water by stating that the two are immiscible fluids separated by sharp interfaces 7, 13]. These are computationally and conceptually simple models, and fail to describe transition zones with strong dispersion and are mathematically intractable in the case of complex topology of the interface [1, 7].

Conversely, density-dependent models solve both the coupled flow equations and transport equations throughout the domain directly finding the concentration gradients and mixing processes [2, 15]. These models represent the full physics of variable-density flow, however, at a high computational cost because the nonlinear coupling between solutes and hydraulic head is represented. The computational requirements are further enhanced with grid refinement requirements at and around interfaces, and numerical problems, particularly in three dimensional domain and non-homogenous materials [15].

Level set methods, which have the inherent capability to handle the dynamics of moving interfaces, which are the zero contour of a scalar field in higher-dimensional space, are a promising method of computational approach to simulating saltwater intrusion [10]. These methods have been developed based on their roots in computer graphics and fluid mechanics and have the ability to deal with complex topological transitions, such as the merging and fragmentation of interfaces mathematically robustly without the use of explicit boundary reconstruction [6, 10]. Despite the above theoretical strengths, the practical aspects of the application of level set methods in managing coastal aquifers are not well studied. Specifically, there is scanty literature in the domain-specific literature on how well they capture the geometry of saltwater-freshwater interface, how well they perform computationally, and their applicability to models that rely on density-driven ground water flow [1, 10]. Such a lack of systematic evaluation prevents the wise choice of methodologies and also limits the adoption of level set methods into real and useful field-scale applications, in which reliably describing saltwater intrusion is essential to water resource management and planning [16].

Research Questions

1. Developing synthetic the conceptual basis of level set methods of saltwater intrusion modeling;
2. The aspects of numerical implementation are to be considered critically;
3. Assessment of replication capacity of Henry problem;
4. Strengths and limitations definition; and
5. Working out the recommendations on methodological advances.

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Theoretical Framework

The level set methods, initially developed by Osher and Sethian (1988) offer a mathematical representation and tracing of the moving interfaces in a domain of computation. The evolving interface, in this method, is implicitly formulated as the zero level set of a scalar valued function whose dimension is higher. This methodology allows the explicit tracking problem to be reduced to a scalar field dynamics, allowing the handling of topological changes of interest (merging and fragmentation) with great accuracy [6, 10].

The mathematical formulation of the motion of moving interfaces was initially proposed by Osher and Sethian (1988) as level set methods [10,17]. The rationale of this strategy is that implicitly the interface can be described as the zero-level set of a scalar function of higher dimension. This value is higher than 0 in one phase and lower than 0 in the other one.

The advection equation controls the development of the interface [10].

$\partial\varphi/\partial t + \mathbf{u}\cdot\nabla\varphi = 0$

and where the field of velocity is denoted by 2. It is a reformulation of a challenging problem of explicitly tracing an explicit boundary to a problem of transport of scalars based on a given spatial domain [10]. Complications that are more complex (merging and discontinuity between interfaces) are automatically handled without special care. Also, the interface normal and curvature may be easily obtained as the direct derivatives of the regular differential operators of a standard or ordinary function, which are due to a calculation of the interface.

Figure 2 shows a workflow breakdown structure of the sequential activities in implementing a level set method of saltwater intrusion modeling. As illustrated, the workflow is a combination of interface init, numerical solution of the equations of groundwater flow and transport, periodic

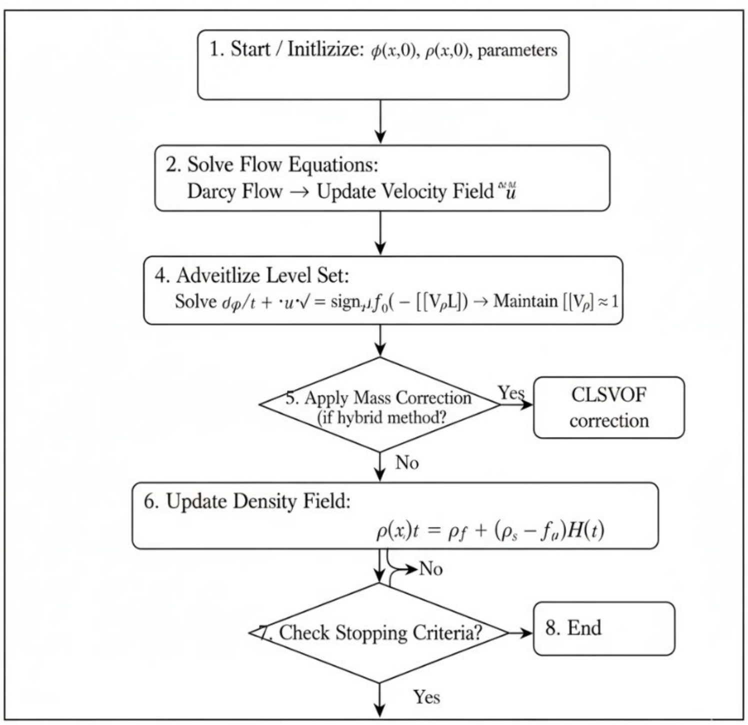

Figure 2. Typical computationl workflow for a lovel lest-based saltwr intrusion model, illustrating the sequecial coupling beupling between the flow solution and interface tracking

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Reinitialisation of the level set function, and mass balance corrections. The schematic codifies the methodological framework addressed in the previous text and demonstrates the algorithmic processes of the iterative character of the interface tracking in the coastal aquifer systems.
Normally, level set functions are reinitialized [19] periodically to maintain such properties of a signed distance functional as the property that the gradient of the function is equal to one, i.e. the union of the points where the function is equal to zero is called the zero level set. Without reinitialisation, the numerical error will lead to errors in the calculation of the interface functions, and the gradient of the functions, 1, will be erroneous to an illegitimate distance functional [10].

External validity to Saltwater Intrusion.
The level set function is applied implicitly to find the dynamic system boundary between freshwater and saltwater as applied in the example of saltwater intrusion in coastal aquifers. The temporal evolution of this interface is governed by the underlying field of Darcy velocity that defines the flow of ground water as an manifestation of spatially and temporarily varying hydraulic gradient and fluid densities. Stated differently, the velocity field u is the resultant of the competing effects of permeability, viscosity, pressure fields, density differences, gravity and height, hence it determines the speed of and the direction of interface flow [7].
$u = -(K/\mu)(\nabla p + \rho g \nabla z)$
And the continuity equation
$\nabla \cdot u + S \partial p/\partial t = Q$
As an example, the use of the gradient flow (VTP) model with the following parameterization, the inclination of an interface is 30 degrees, a process coefficient Cp is 0.015, and a storage coefficient S is 10. Here, the source or sink term Q is calculated whereby; Q = -2 [-1 + 0.6] which is the required angular and model parameters. The values obtained as part of the normalization process of the VTP model are used to illustrate the methodological approach and sensitivity of parameters to the selection of the technique.
Given the intrinsic permeability, dynamic viscosity, pressure, fluid density, acceleration due to gravity, elevation, specific storage, and sources or sinks, and represented by, respectively, K, m, p, rho, g, z, S and Q [1, 7].
The difference in the interface densities are incorporated by a smoothed version of the Heaviside functional:
$\rho(x,t) = \rho f + (\rho s - \rho f)H(\varphi)$
Where, rho f, and rho s are density of fresh and saltwater respectively [1]. The Heaviside function is a regularized version of a transition band, with numerically computed solutions, and, however, it has rather sharp transitions when the width is selected with respect to the grid spacing.

There are several modeling options that define the formulation of saltwater intrusion models that are level set based, each of which has a direct impact on the consequences of the simulations and their computational requirements. First, modelers must make a choice to solve a unified field of the Darcy velocity in the whole domain, or to solve segregated fields of freshwater and saltwater domains; the segregated field fields expressly capture the difference between freshwater and saltwater flow using different hydraulic conductivity and density but are more complicated [1, 7]. Second, phase-based assignments of representation of density variations can be discrete, i.e., in discrete representations of the phase or continuous, i.e. in the smoothed Heaviside formulations of the level set variable which influence smoothness of the solution and sharpness of the interface. Thirdly, whether or not the level set evolution equation contains dispersive and diffusive terms should be considered i.e. explicit hydrodynamic dispersion may be added to improve the representation of mixing in the transition zone

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between freshwater and saltwater [1, 14]. Finally, the numerical realization needs the selection of an algorithm sequence to advance the level set function, velocity, pressure and, in case of application, the respective width fields to pay attention to. It is typically founded on iterative or operator-splitting schemes, in order to reach convergence and stability, each iterative coupling scheme is explicitly, semi-implicitly or fully implicit, and has various trade-offs in the form of the cost of computation and robustness [1, 2].

Numerical Implementation

Temporal and Spatial Discretization.

Classical second order central-difference estimates of the advection term.

$\varphi(1) = \varphi n + \Delta tL(\varphi n)$

$\varphi(2) = (3/4)\varphi n + (1/4)\varphi(1) + (1/4)\Delta tL(\varphi(1))$

$\varphi n+1 = (1/3)\varphi n + (2/3)\varphi(2) + (2/3)\Delta tL(\varphi(2))$

High order weighted essentially non-oscillatory (WENO) schemes, in particular, are especially simple to apply to level set transport, since they apply smooth high order stencils in smooth regions, and minimize oscillations in discontinuities [10].

The technique of the third-order total variation decreasing (TVD) Runge-Kutta approach is generally employed in order to carry out the temporal integration:

Where L is the spatial discretization operator. The scheme is able to maintain normal CFL stability and have a third-order stability with respect to time [1].

Table 2 — Key numerical parameters and their impact on level set model performance

Parameter	Typical Range	Impact on Model Performance
Grid resolution	50–200 cells / characteristic length (problem dependent)	Higher resolution improves interface accuracy and reduces numerical diffusion (can approach 1st–2nd order convergence), but increases CPU/memory cost.
Reinitialization frequency	Every 5–20 time steps	Too frequent → artificial mass loss; too infrequent → level set loses signed-distance property and accuracy degrades. Choose frequency based on CFL and advection error.
Transition band width (ε)	~1.5–3 × grid spacing	Controls interface sharpness vs. stability. Smaller ε → sharper interface but higher sensitivity to grid noise; larger ε → smoother, more stable interface but diffused toe.
Time step (CFL number)	CFL < 1.0 (problem dependent; often 0.2–0.8)	Smaller time steps increase stability and accuracy of level set advection (reduce overshoot/undershoot) but increase runtime. Respect coupling constraints with flow solver.
Coupling strategy (flow ↔ interface)	Explicit → Iterative / implicit	Explicit coupling is cheaper and acceptable for low density contrast or steady flow; iterative/implicit coupling improves stability and mass conservation for high density contrasts or transient forcing.

Association Strategies and Reinitialisation.

Darcy flow and level set transport have four major strategies. The current density field implicitly solves the flow equation and then the flow equation is further advanced with the level set [1]. This is also computationally energy efficient but may be disadvantageous in terms of stability where large variations in density are concerned. Sequential semi-implicit coupling, iterative coupling and projection-method coupling are increasingly becoming implicit and therefore more expensive in terms of computational cost, as well as being more stable [1, 2, 6].

The solution to the equation is a signed distance functional, which is maintained by reinitialisation.

$\partial\varphi/\partial\tau = \mathrm{sign}(\varphi 0)\ (1 - |\nabla\varphi|)$

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In which the level set function prior to reinitialisation is represented by a level set function ( ), and pseudo time variable ( ) is represented by the variable ( ). The distinction between the conservative level set methods is that they limit reinitialisation in a manner that ensures that the integral of level set functional is preserved over the domain range [18-20]. In hybrid methods using level sets and volume-of-fluid (CLSVOF), to an existing level set function, volume fractions are explicitly corrected by volume-based data, in order to eliminate mass-balance errors [21].

Henry Problem Problems: Performance.
The issue to be discussed by Henry is the gradual flooding of saltwater in a small coastal aquifer that has the dimensions of 2m by 1m long and height respectively [9]. The freshwater is flowing along the inland boundary of the boundary with a uniform and a given flux and the seaward boundary is subjected to the hydrostatic saltwater pressure. Henry (1959) developed a semi analytical solution, which is composed of a Fourier series expansion [9]. Subsequent work identified a few minor errors in the first solution, and the current use of high-resolution and density-relevant models has been considered a standard [3, 4].

When the Henry problem is solved with level set methods, in combination with high-order WENOs, and with numerical parameters optimized, the positions of the steady-state simulation interfaces agree with established benchmark solutions within 23 percent. Such benchmark inaccuracies are not unique to the literature of other high-fidelity numerical methods, and the ability of level set formulations to provide physically plausible solutions to canonical saltwater intrusion problems makes the error tolerable [3, 4]. Moreover, through a sound choice of parameters, the inland toe modeling error of the saltwater interface is less than 5 percent and this is in general acceptable to use as a validation and is in line with standard uncertainties in groundwater modeling of the coast lines [1].

Mass balance: The loss of volume to steady state will be typically 5-15 percent in pure level set approaches with time [1]. The hybrid CLSVOF techniques reduce these errors by more than 2 percent of the mass-balance errors of conservative finite-volume methods.

Transient cases Transient cases have special advantages with level set formulations, when the conditions on the boundary vary with time [1]. An implicit interface model naturally gives interface motion without algorithmic changes needed by sharp treatment of interface topology when changing topology. The level set methods have been discovered to scale the time scales and spatial distributions of interface changes in consistent agreement with the density-dependent methods with order of magnitude predictions in 10 percent of the most normal parameter space [1, 2].

The summary of systematic sensitivity analysis (Table 2) helps detect a set of important numerical parameters that play a vital role in the work of the model. It has been shown that increasing the spatial resolution, i.e. grid densities of at least 50 cells/m, can significantly increase the accuracy of interface positions, and convergence rates of between the first and second order have been observed [1]. Reinitialisation frequency is also critical; a good trade-off between the signed distance property of the level set function and cumulative mass loss is a compromise between reinitialisation frequency every 5 to 20 time steps. Also, the choice of the regularization bandwidth has a strong control over interface sharpness and numerical stability. It is worth noting that when the time step of the simulation is reduced below the regularization bandwidth, then stabilizing effects can be compromised which can lead to deteriorated solution fidelity. These results, tabulated in Table 2, allow us to suggest numerical parameters that can achieve a compromise between accuracy, computational efficiency and mass

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conservation in level set-based modeling of saltwater intrusion. Figure 1 presents a schematic of the Henry problem benchmark, which serves as the foundational configuration for validating saltwater intrusion models.

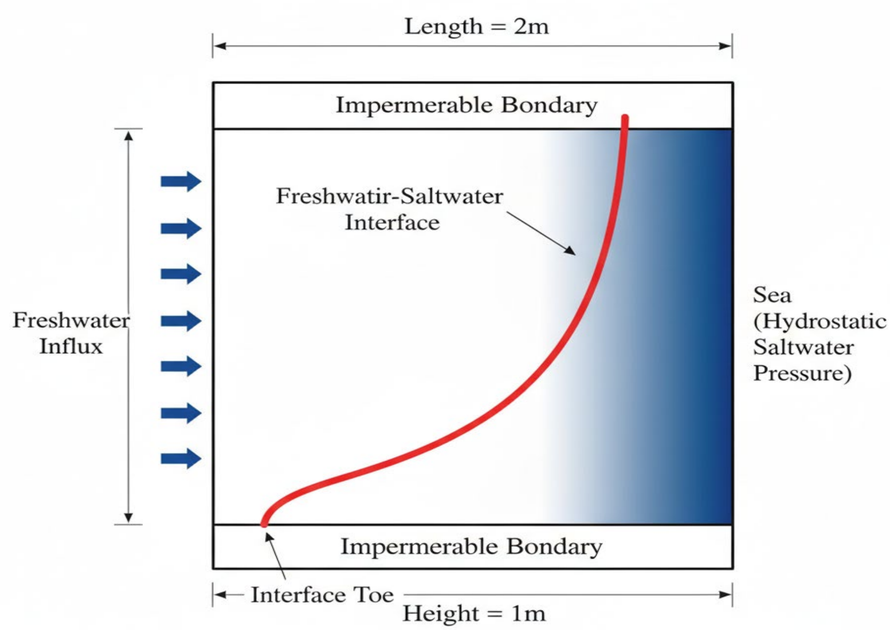

Figure 1. Schematic diagram of the classical Henry's Problem for saltwatte intrustion. The confined aquiler hass ia length of 2m boundary, while hyalrstatic saltwatter pressure, sot of the presse onta feighh bandary. The top and botom are interm ite "being its ints-point [25].

As referenced in the text, this diagram depicts the idealized two-dimensional coastal aquifer domain, indicating critical boundary conditions, including uniform freshwater inflow at the inland boundary and hydrostatic saltwater pressure at the seaward boundary. The figure further annotates essential geometric and hydraulic characteristics to elucidate the spatial arrangement and principal flow dynamics, which are crucial for rigorous model assessment.

Complex Systems Extrapolation.

Anisotropic Dispersion

In Anisotropic Henry problem, tensrs of dispersion are employed where the longitudinal component is much larger than the transverse component [14]. Level-set approaches have seldom been considered with this extension in mind. With the inclusion of isotropic diffusion terms, the evolution equation of the level-set fails to anisotropically mix anisotropic media [1].

Anisotropic diffusion applied to the level-set framework, combining the level-set dynamics with the concentration transport, and post-hoc construction of the transition zones are some of the potential solutions [22]. Tight testing of such schemes when the anisotropic Henry strategies are known is research objective [14].

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Hydraulic Conductivity, anisotropic.
The possibilities and difficulties of extension to heterogeneous aquifer systems in which the hydraulic conductivity varies spatially [7, 15]. The implicit interface representation bypasses the explicit tracing of interface-heterogeneity intersections, and therefore it can potentially be of benefit when compared to the sharp-interface schemes. Initial tests of basic heterogeneous systems indicate that formulations of level-sets can support the variability of conductivity without necessitating any fundamental modifications to its associated algorithms [23].

However, some aspects are to be noted. The heterogeneous media are likely to require the finer discretization in order to resolve the geometry of interfaces and the geological structure [1]. There are a number of macro-dispersion due to heterogeneity of the velocity fields which need appropriate representation. Heterogeneity can have an adverse impact on reinitialisation by the complexity that it imposes on the interface. In addition to that, the heterogeneous saltwater intrusion problem has a lack of benchmark solutions, which limits the validation of the level-set methods to such environments [14].

Comparative Assessment

Computational Effectiveness and Interface Solution.
At moderate grid resolutions of two-dimensional problems, level-set methods typically cost 6080 percent of the computational cost of similar resolution density-dependent models when explicitly coupled together [10-12]. This advantage rests on the fact that the flow equation is solved, unlike a coupled flow-transport system [2]. The benefit of growing the size of the problem being resolved or dimensionality grows smaller the larger the problem size because of the stability limits of CFL and reinitialisation expenses [13].

The most crucial measures of performance are the sharpness of the interface and the precision of position. High-order WENO schemes used as level-set algorithms achieve interface resolution of 3-5 grid cell resolution (as compared to density-dependent schemes of the same spatial discretization [7]. On problems that are steady-state and sharp-interface, sharp-interface assumptions can be applied to provide the most accurate interface positions by sharp-interface methods [7, 13]. Level-set methods are intermediate, as they provide the right location of the interfaces, but they are extremely sparse in the information about the distributions of concentrations across the transition plane.

Flexibility and Mass Conservation Geometric.
The big advantage of the level-set formulations is the geometrical flexibility [1]. The morphologies of higher-dimensional interfaces, including overhanging morphologies and topology topography are modeled directly, without specialized algorithmic processing [6]. In problems with multiple interfaces at the same time, level-set algorithms can be extended to a sequence of level-set functions, and one of these functions is the level-set volume of any given interface [10].

One of the major weaknesses of traditional level set techniques is that they are not very able to preserve mass over the simulated domain [1]. The loss of artificial phase volume and errors that accrue with time are caused by numerical diffusion together with the necessity to periodically reinitialise to preserve the signed distance property of the level set function. Empirical research shows that conventional level set models will tend to have steady-state volumetric deficits of between 5 and 15 per cent and this compromises long-term saltwater intrusion modelling. Enhanced approaches have been designed to aid conservation of masses by use of algorithms such as conservative reinitialisation

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algorithms which limit deviations of the property of signed distance and the loss of unwanted phases. In addition, hybrid schemes, e.g., the coupled conservative level set/volume-of-fluid (CLSVOF) framework, explicitly control the phase volumes by directly taking volumetric data into account in the interface-capturing scheme [1]. Other remediation schemes such as Lagrangian particle-level corrections and global volume correction also minimize the net mass loss by imposing a systematic systemic correction of deficits at either the local or global level. In the simulations of CLSVOF using cumulative volumetric errors, with such improvements, one can achieve a level of less than 2% error in cumulative volumetric errors, which significantly improves accuracy and reliability of the interface representation in saltwater intrusion models.

Discussion and Conclusions

Table 1 provides a structured comparison of the principal features, advantages, and constraints of sharp-interface, density-dependent, and level set methods as applied to saltwater intrusion modeling.

Feature	Sharp-Interface Models	Density-Dependent Models	Level Set Methods
Interface Representation	Explicit boundary	Implicit via concentration field	Implicit via level set function φ
Mixing / Transition Zone	Neglected	Explicitly resolved	Limited representation; can be added (e.g., via smoothing/regularization)
Computational Cost	Low	High	Moderate
Geometric Flexibility	Low (complex to track topology changes)	High (fixed grid, handles gradients)	Very high (handles topology changes and merging/splitting)
Mass Conservation	Inherent per-domain (depends on implementation)	Inherent (with good numerics)	Poor by default (requires correction/hybrid schemes)
Primary Strength	Computational efficiency for advection-dominated problems	Physical fidelity (mixing, dispersion, density coupling)	Handles complex, evolving interfaces and topology robustly
Primary Limitation	Cannot model dispersion; tracking complex topology is hard	High computational demand and stricter stability constraints	Mass conservation issues, parameter sensitivity, needs reinitialization/correction

The table explicitly details differences in interface representation precision, capacity for geometric adaptation, numerical efficiency, and the degree of mass conservation achieved by each technique. Furthermore, Table 1 directly addresses the methodological suitability of these approaches for simulating complex interface dynamics and sustaining accuracy over long-term scenarios. This synthesis offers a critical reference point for researchers and practitioners selecting appropriate modeling frameworks for coastal aquifer analyses.

Level set methods are a sound and technically feasible approach to the simulation of saltwater intrusion which has been demonstrated by their capacity to provide benchmark solutions to problems including the Henry problem [1, 3, 9]. Comparative analyses show that the interface positions by the formulations of high-fidelity level sets are in close agreement with the results of well-known density-dependent models and are within the acceptable uncertainty bands of field-scale applications [1, 3, 4]. It is interesting to note that the implicit interface representation of the level set methods provides a great deal of geometric freedom allowing modeling of complex interface geometries, such as convoluted or

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discontinuous interface boundaries, with great accuracy. This geometrical flexibility is in contrast with either the more constrained nature of sharp-interface and more classical density-dependent methods, which either need to explicitly rebuild interfaces or are less able to support dynamic topological changes, e.g., combining or splitting. When dealing with the changing or disjointed interfaces, the level set approaches are better placed to provide the automated tracking of interfaces, and this aspect highlights why the approach is appropriate when dealing with complex saltwater intrusion processes [6, 7, 10]. But the mass conservation problems are known to impede the eager application of traditional level-set techniques in long-term predictive modeling. Mass-conservation corrections are necessary in the formulation of base formulations, which complicates and cancels the benefits of computational efficiency to some extent [5].

The computational performance tests are showing that, with two-dimensional problems of moderate spatial resolution, level set methods provide a modest improvement in computational efficiency over fully density-dependent formulations [16]. The geometric flexibility of level set methods makes them especially adapted to exploring problems with complicated morphologies of interfaces and performing robust uncertainty quantification, e.g. by large-scale Monte Carlo sampling. Nonetheless, in applications where mass conservation is important, or where the concentration of solutes in transition zones needs a high degree of resolution, e.g. in regulatory modelling, infrastructure planning or ecological risk assessment, standard level set formulations might provide unacceptable error unless hybrid approaches that conserve mass are employed. Thus interface tracking methodology needs to be chosen based on the actual real-world goals of the modeling endeavor. Error in the mass balance or interface definition may carry major errors in the water resource apportionment, prediction of contaminant transport, and the formulation of the climate adaptation solutions [16].

The identified priority research directions cover the following gaps, which are core issues in the implementation and development of level set methods to model saltwater intrusion. First, there should be field-scale validation experiments to test model behavior under real-life conditions and, therefore, to make the results of the simulation credible. Second, the systematic extension of benchmark problem suites to include the heterogeneities and anisotropy of aquifers will help to achieve a sound model testing and validation in a variety of realistic situations [14, 15, 16]. Third, to enhance mass conservation and interface sharpness, hybrid level set/volume-of-fluid methods should also be developed to facilitate the solution of complex or long-term problems. Fourth, the combination of the level set frameworks with the uncertainty quantification tool will facilitate the rigorous assessment of risks and inform the water resource management decisions in the presence of the parametric uncertainty. Fifth, the development of effective and precise volume-of-fluid algorithms in three dimensional spaces is important in order to provide the entire three dimensional image of aquifer systems [15, 16]. Lastly, the modeling of the dispersion processes needs to be developed further in order to effectively reflect the mechanism of solute transport in transition zones. To encourage wide use and replication, it is evident that comprehensive, validated and open-source software implementations are to be published that are compatible with well-established groundwater modeling platforms [1,24].

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